Advanced Topics - Deep Learning Basics

Introduction to Deep Networks, Activation Functions, and Common Architectures

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Understanding Deep Learning

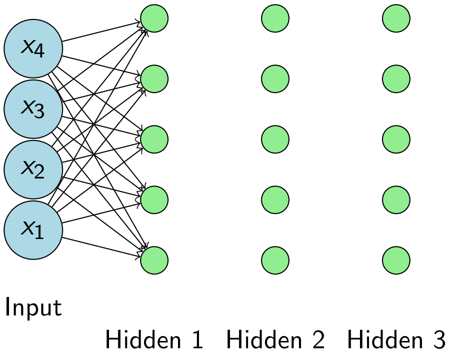
Today's Learning Journey

What is Deep Learning?

Deep Learning is a subset of machine learning that uses artificial neural networks with multiple layers (deep networks) to model and understand complex patterns in data.

Key Characteristics:

- Multiple hidden layers (typically 3+ layers)
- Automatic feature extraction
- Non-linear transformations
- End-to-end learning



Deep Learning vs Traditional ML

Traditional Machine Learning

- Manual feature engineering
- Shallow learning algorithms
- Limited representation power
- Good for structured data
- Interpretable models

Examples: Linear Regression, SVM, Decision Trees, Random Forest

Deep Learning

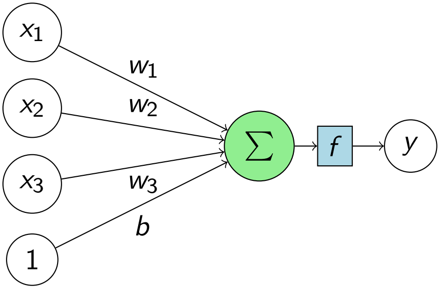
- Automatic feature learning
- Deep neural networks
- High representation power
- Excellent for unstructured data
- Black-box models

Examples: CNNs, RNNs, Transformers, GANs

The Perceptron: Building Block

Single Perceptron:

$$
\begin{aligned}
y &= f\left(\sum_{i=1}^{n} w_i x_i + b\right) && (1)\\
&= f(\mathbf{w}^T\mathbf{x} + b) && (2)
\end{aligned}
$$



Where:

- $\mathbf{x}$: input vector
- $\mathbf{w}$: weight vector
- b: bias term
- f: activation function

Multi-Layer Perceptron (MLP)

Forward Propagation:

$$\mathbf{h}^{(1)} = f^{(1)}(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}) \tag{3}$$

$$\mathbf{h}^{(2)} = f^{(2)}(\mathbf{W}^{(2)}\mathbf{h}^{(1)} + \mathbf{b}^{(2)}) \tag{4}$$

$$\vdots \tag{5}$$

$$\mathbf{y} = f^{(L)}(\mathbf{W}^{(L)}\mathbf{h}^{(L-1)} + \mathbf{b}^{(L)}) \tag{6}$$

Where:

- $\mathbf{h}^{(l)}$: hidden layer l activations
- $\mathbf{W}^{(l)}$: weight matrix for layer l
- $\mathbf{b}^{(l)}$: bias vector for layer l
- $f^{(l)}$: activation function for layer l
- L: total number of layers

Why Activation Functions?

Purpose of Activation Functions:

- Introduce **non-linearity** into the network
- Enable learning of complex patterns
- Without activation functions, deep networks collapse to linear models

Mathematical Insight:

$$\begin{aligned}
\text{Without activation: } \mathbf{y} &= \mathbf{W}^{(2)}(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}) + \mathbf{b}^{(2)} && (7)\\
&= \mathbf{W}^{(2)}\mathbf{W}^{(1)}\mathbf{x} + \mathbf{W}^{(2)}\mathbf{b}^{(1)} + \mathbf{b}^{(2)} && (8)\\
&= \mathbf{W}'\mathbf{x} + \mathbf{b}' \quad \text{(Linear transformation)} && (9)
\end{aligned}$$

Common Activation Functions

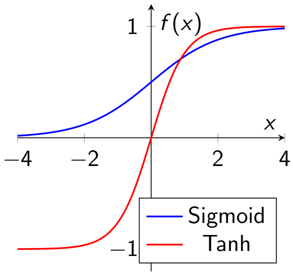
1. Sigmoid Function $\sigma(x) = \frac{1}{1+e^{-x}}$

Properties:

- Range: $(0, 1)$
- Smooth, differentiable
- Prone to vanishing gradients

2. Hyperbolic Tangent (Tanh)

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \tag{10}$$



Properties:

- Range: $(-1, 1)$
- Zero-centered
- Still suffers from vanishing gradients

3. ReLU (Rectified Linear Unit)

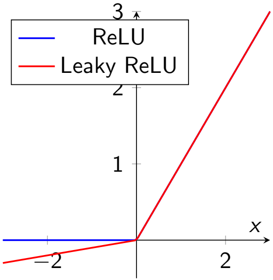
$$\mathrm{ReLU}(x) = \max(0, x) \tag{11}$$

Advantages:

- Simple computation
- No vanishing gradient problem
- Sparse activation

Disadvantages:

- Dying ReLU problem
- Not differentiable at $x = 0$



4. Leaky ReLU

$$\mathrm{LeakyReLU}(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha x & \text{if } x \leq 0 \end{cases} \tag{12}$$

5. Swish/SiLU (Sigmoid Linear Unit)

$$\text{Swish}(x) = x \cdot \sigma(x) = \frac{x}{1 + e^{-x}} \tag{13}$$

6. GELU (Gaussian Error Linear Unit)

$$\text{GELU}(x) = x \cdot \Phi(x) = \frac{x}{2}[1 + \text{erf}(\frac{x}{\sqrt{2}})] \tag{14}$$

Choosing Activation Functions:

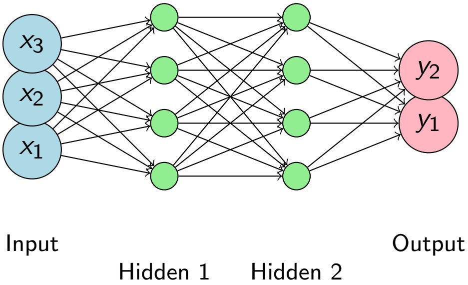
- **Hidden layers:** ReLU (default choice), Swish, GELU
- **Output layer:**
  - Binary classification: Sigmoid
  - Multi-class classification: Softmax
  - Regression: Linear (no activation)

Feedforward Neural Networks

Architecture: Information flows in one direction from input to output

Characteristics:

- Fully connected layers
- No cycles or loops
- Each neuron connects to all neurons in next layer
- Universal function approximators



Applications:

- Classification tasks
- Regression problems
- Feature learning
- Function approximation

Convolutional Neural Networks (CNNs)

Designed for processing grid-like data (images, time series)

Key Components:

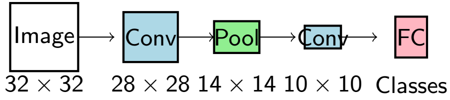
- **Convolutional layers:** Feature extraction
- **Pooling layers:** Dimensionality reduction
- **Fully connected layers:** Classification

Advantages:

- Translation invariance
- Parameter sharing
- Local connectivity
- Hierarchical feature learning

Applications:

- Image classification, Object detection
- Medical imaging, Computer vision



Convolution Operation:

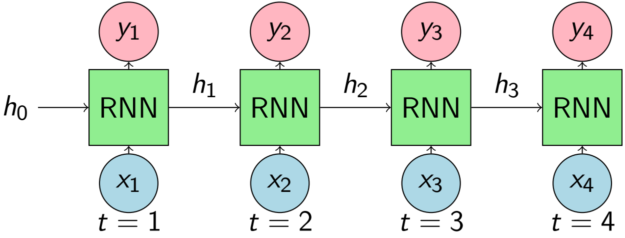
$$(f * g)(i, j) = \sum_m \sum_n f(m, n) \cdot g(i - m, j - n) \tag{15}$$

Recurrent Neural Networks (RNNs)

Designed for sequential data processing

Key Characteristics:

- Memory through hidden states
- Process sequences of variable length
- Share parameters across time steps
- Can model temporal dependencies



RNN Equations:

$$\mathbf{h}_t = f(\mathbf{W}_{hh}\mathbf{h}_{t-1} + \mathbf{W}_{xh}\mathbf{x}_t + \mathbf{b}_h) \tag{16}$$

$$\mathbf{y}_t = \mathbf{W}_{hy}\mathbf{h}_t + \mathbf{b}_y \tag{17}$$

Variants:

- **LSTM:** Long Short-Term Memory
- **GRU:** Gated Recurrent Unit

Applications:

- Natural Language Processing
- Time series forecasting
- Speech recognition
- Machine translation

Backpropagation Algorithm

Core idea: Compute gradients of loss function with respect to network parameters using chain rule

Forward Pass: Compute predictions

$$\mathbf{a}^{(l)} = f^{(l)}(\mathbf{z}^{(l)}) \quad \text{where} \quad \mathbf{z}^{(l)} = \mathbf{W}^{(l)}\mathbf{a}^{(l-1)} + \mathbf{b}^{(l)} \tag{18}$$

Backward Pass: Compute gradients

$$\delta^{(l)} = \frac{\partial \mathcal{L}}{\partial \mathbf{z}^{(l)}} \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(l)}} = \delta^{(l)}(\mathbf{a}^{(l-1)})^T \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{b}^{(l)}} = \delta^{(l)} \tag{21}$$

$$\delta^{(l-1)} = (\mathbf{W}^{(l)})^T\delta^{(l)} \odot f'^{(l-1)}(\mathbf{z}^{(l-1)}) \tag{22}$$

Where $\mathcal{L}$ is the loss function and $\odot$ denotes element-wise multiplication.

Gradient Descent Optimization

Parameter Update Rule:

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta \mathcal{L}(\theta_t) \tag{23}$$

Where η is the learning rate and θ represents all network parameters.

Variants:

- **Batch Gradient Descent:** Uses entire dataset
- **Stochastic Gradient Descent (SGD):** Uses single examples
- **Mini-batch Gradient Descent:** Uses small batches

Advanced Optimizers:

- **Adam:** Adaptive moment estimation
- **RMSprop:** Root mean square propagation
- **AdaGrad:** Adaptive gradient algorithm

Vanishing and Exploding Gradients

Vanishing Gradients:

- Gradients become very small in early layers
- Common with sigmoid/tanh activations
- Deep networks fail to learn

Solutions:

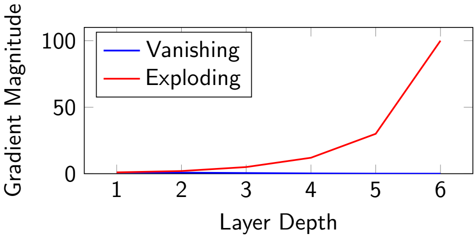
- Use ReLU activations
- Proper weight initialization
- Batch normalization
- Residual connections

Exploding Gradients:

- Gradients become very large
- Unstable training
- Parameters oscillate wildly

Solutions:

- Gradient clipping
- Proper weight initialization
- Lower learning rates
- Batch normalization



Overfitting in Deep Networks

Overfitting: Model performs well on training data but poorly on unseen data

Causes:

- Too many parameters
- Insufficient training data
- Complex model architecture
- Training for too long

Detection:

- Large gap between training and validation loss
- Validation accuracy decreases while training accuracy increases

Regularization Techniques:

- **Dropout:** Randomly set neurons to zero
- **L1/L2 Regularization:** Add penalty terms
- **Early Stopping:** Stop when validation loss increases
- **Data Augmentation:** Increase training data
- **Batch Normalization:** Normalize layer inputs

Dropout Mathematical Formulation:

$$\tilde{\mathbf{h}} = \mathbf{r} \odot \mathbf{h} \quad \text{where} \quad r_i \sim \text{Bernoulli}(p) \tag{24}$$

During training: p is dropout probability. During inference: scale by $(1 - p)$.

Batch Normalization

Problem: Internal covariate shift - distribution of layer inputs changes during training

Batch Normalization Algorithm:

$$\mu_B = \frac{1}{m}\sum_{i=1}^{m} x_i \quad \text{(Batch mean)} \tag{25}$$

$$\sigma_B^2 = \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_B)^2 \quad \text{(Batch variance)} \tag{26}$$

$$\hat{x}_i = \frac{x_i - \mu_B}{\sqrt{\sigma_B^2 + \epsilon}} \quad \text{(Normalize)} \tag{27}$$

$$y_i = \gamma \hat{x}_i + \beta \quad \text{(Scale and shift)} \tag{28}$$

Where γ and β are learnable parameters, ϵ is a small constant for numerical stability.

Benefits:

- Faster training convergence, Higher learning rates possible
- Reduces internal covariate shift, Acts as regularization

Skip Connections and ResNet

Residual Learning: Learn residual mapping instead of direct mapping

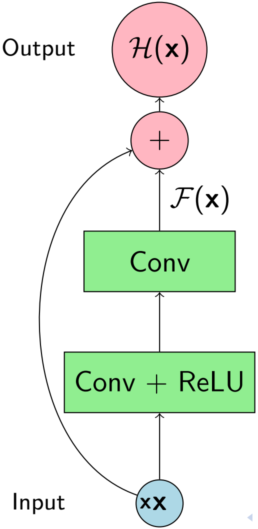
Traditional Network:

$$\mathcal{H}(\mathbf{x}) = \text{desired mapping} \tag{29}$$

Residual Network:

$$\mathcal{H}(\mathbf{x}) = \mathcal{F}(\mathbf{x}) + \mathbf{x} \tag{30}$$

$$\mathcal{F}(\mathbf{x}) = \mathcal{H}(\mathbf{x}) - \mathbf{x} \tag{31}$$



Advantages:

- Solves vanishing gradient problem
- Enables very deep networks (100+ layers)
- Identity mapping when $\mathcal{F}(\mathbf{x}) = 0$
- Improved gradient flow

Attention Mechanisms

Motivation: Focus on relevant parts of input sequence

Attention Formula:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V \tag{32}$$

Where:

- Q: Query matrix
- K: Key matrix
- V: Value matrix
- d_k: Dimension of key vectors

Self-Attention: Q, K, and V are all derived from the same input sequence

Applications:

- **Transformer Architecture:** GPT, BERT, T5
- **Computer Vision:** Vision Transformers (ViTs)
- **Multimodal:** CLIP, DALL-E

Weight Initialization

Why Important: Poor initialization can lead to vanishing/exploding gradients

Common Initialization Methods:

- **Xavier/Glorot Initialization:**

$$W \sim \mathcal{N}\left(0, \frac{2}{n_{in} + n_{out}}\right) \tag{33}$$

- **He Initialization (for ReLU):**

$$W \sim \mathcal{N}\left(0, \frac{2}{n_{in}}\right) \tag{34}$$

- **LeCun Initialization:**

$$W \sim \mathcal{N}\left(0, \frac{1}{n_{in}}\right) \tag{35}$$

Where n_{in} is number of input units and n_{out} is number of output units.

Rule of Thumb: Use He initialization for ReLU networks, Xavier for sigmoid/tanh networks.

Hyperparameter Tuning

Key Hyperparameters in Deep Learning:

Architecture:

- Number of layers
- Number of neurons per layer
- Activation functions
- Network topology

Training:

- Learning rate
- Batch size
- Number of epochs
- Optimizer choice

Regularization:

- Dropout rate
- L1/L2 regularization strength
- Early stopping patience
- Data augmentation parameters

Tuning Strategies:

- Grid search
- Random search
- Bayesian optimization
- Automated ML (AutoML)

Best Practice: Start with established architectures and fine-tune from there.

Model Evaluation and Validation

Evaluation Strategies:

- **Train/Validation/Test Split:** 60
- **Cross-Validation:** K-fold cross-validation
- **Hold-out Validation:** Simple train/test split

Metrics for Different Tasks:

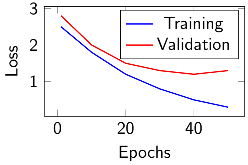
Classification:

- Accuracy
- Precision, Recall, F1-score
- ROC-AUC
- Confusion Matrix

Regression:

- Mean Squared Error (MSE)
- Mean Absolute Error (MAE)
- R-squared (R^2)
- Root Mean Squared Error (RMSE)

Learning Curves:



Real-World Applications

Computer Vision:

- Image classification
- Object detection
- Facial recognition
- Medical imaging
- Autonomous driving

Natural Language Processing:

- Machine translation
- Sentiment analysis
- Chatbots and virtual assistants
- Text summarization
- Question answering

Other Domains:

- **Speech:** Recognition, synthesis
- **Robotics:** Control, navigation
- **Gaming:** AlphaGo, game AI
- **Finance:** Fraud detection, trading
- **Healthcare:** Drug discovery, diagnosis
- **Art:** Style transfer, generation

Emerging Applications:

- Climate modeling
- Protein folding
- Scientific discovery
- Creative content generation

Current Trends and Future Directions

Current Hot Topics:

Architecture Innovations:

- **Transformers:** Attention is all you need
- **Vision Transformers:** ViT, DeiT, Swin
- **Efficient Architectures:** MobileNet, EfficientNet
- **Neural Architecture Search:** AutoML for architectures

Training Innovations:

- Self-supervised learning
- Few-shot learning
- Meta-learning
- Continual learning

Large-Scale Models:

- Large Language Models (GPT, BERT)
- Foundation models
- Multimodal models
- Scaling laws

Challenges and Opportunities:

- Interpretability and explainability
- Robustness and adversarial attacks
- Energy efficiency
- Fairness and bias
- Democratization of AI

Key Concepts Review

What We Covered Today:

1. **Deep Learning Fundamentals:**
   - Multi-layer neural networks
   - Forward and backward propagation
   - Universal function approximation
2. **Activation Functions:**
   - ReLU family (ReLU, Leaky ReLU)
   - Traditional functions (Sigmoid, Tanh)
   - Modern variants (Swish, GELU)
3. **Common Architectures:**
   - Feedforward Networks (MLPs)
   - Convolutional Neural Networks (CNNs)
   - Recurrent Neural Networks (RNNs)
4. **Training Challenges and Solutions:**
   - Vanishing/exploding gradients
   - Overfitting and regularization
   - Modern techniques (BatchNorm, ResNet, Attention)

Best Practices Summary

Architecture Design:

- Start simple, then add complexity
- Use proven architectures as baselines
- Consider computational constraints
- Match architecture to problem type

Training Strategy:

- Use appropriate initialization
- Monitor training/validation curves
- Apply regularization techniques
- Use modern optimizers (Adam, AdamW)

Debugging and Optimization:

- Overfit on small dataset first
- Check gradient magnitudes
- Visualize learned features
- Use learning rate schedules

Evaluation:

- Use proper train/validation/test splits
- Report multiple metrics
- Consider domain-specific evaluation
- Test on diverse datasets

Next Steps in Your Deep Learning Journey

Immediate Next Steps:

- Implement basic neural networks from scratch
- Experiment with different activation functions
- Try various optimization algorithms
- Practice with real datasets

Advanced Topics to Explore:

- **Specialized Architectures:** Transformers, GANs, VAEs
- **Advanced Training:** Transfer learning, multi-task learning
- **Optimization:** Learning rate scheduling, gradient clipping
- **Deployment:** Model compression, quantization, edge deployment

Recommended Resources:

- Deep Learning by Ian Goodfellow, Yoshua Bengio, Aaron Courville
- Papers With Code (paperswithcode.com)
- PyTorch/TensorFlow tutorials
- Coursera Deep Learning Specialization

Thank You!

Questions and Discussion

Remember: Deep learning is both an art and a science.
Practice, experiment, and stay curious!